

Exercises 4.4

In Exercises 1–6, find the domain of the function.

1. $f(x) = \frac{-3x}{2x+5}$

2. $g(x) = \frac{x^3 + x + 1}{2x^2 - 5x - 3}$

3. $h(x) = \frac{6x-5}{x^2-6x+4}$

4. $g(x) = \frac{x^3 - x^2 - x - 1}{x^5 - 36x}$

5. $f(x) = \frac{x^5 - 2x^3 + 7}{x^3 - x^2 - 2x + 2}$

6. $h(x) = \frac{x^5 - 5}{x^4 + 12x^3 + 60x^2 + 50x - 125}$

In Exercises 7–12, use algebra to determine the location of the vertical asymptotes and holes in the graph of the function.

7. $f(x) = \frac{x^2 + 4}{x^2 - 5x - 6}$

8. $g(x) = \frac{x-5}{x^3 + 7x^2 + 2x}$

9. $f(x) = \frac{x}{x^3 + 2x^2 + x}$

10. $g(x) = \frac{x}{x^3 + 5x}$

11. $f(x) = \frac{x^2 - 4x + 4}{(x+2)(x-2)^3}$

12. $h(x) = \frac{x-3}{x^2 - x - 6}$

In Exercises 13–22, find the horizontal or other asymptote of the graph of the function when $|x|$ is large, and find a viewing window in which the ends of the graph are within 0.1 of this asymptote.

13. $f(x) = \frac{3x-2}{x+3}$

14. $g(x) = \frac{3x^2 + x}{2x^2 - 2x + 4}$

15. $h(x) = \frac{5-x}{x-2}$

16. $f(x) = \frac{4x^2 - 5}{2x^3 - 3x^2 + x}$

17. $g(x) = \frac{5x^3 - 8x^2 + 4}{2x^3 + 2x}$

18. $h(x) = \frac{8x^5 - 6x^3 + 2x - 1}{0.5x^5 + x^4 + 3x^2 + x}$

19. $f(x) = \frac{x^3 - 1}{x^2 - 4}$

20. $g(x) = \frac{x^3 - 4x^2 + 6x + 5}{x-2}$

21. $h(x) = \frac{x^3 + 3x^2 - 4x + 1}{x+4}$

22. $f(x) = \frac{x^3 + 3x^2 - 4x + 1}{x^2 - x}$

Find x and y intercepts

In Exercises 23–50, analyze the function algebraically: list its vertical asymptotes, holes, and horizontal asymptote. Then sketch a complete graph of the function.

23. $f(x) = \frac{1}{x+5}$

24. $q(x) = \frac{-7}{x-6}$

25. $k(x) = \frac{-3}{2x+5}$

26. $g(x) = \frac{-4}{2-x}$

27. $f(x) = \frac{3x}{x-1}$

28. $p(x) = \frac{x-2}{x}$

29. $f(x) = \frac{2-x}{x-3}$

30. $g(x) = \frac{3x-2}{x+3}$

31. $f(x) = \frac{1}{x(x+1)^2}$

32. $g(x) = \frac{x}{2x^2 - 5x - 3}$

33. $f(x) = \frac{x-3}{x^2 + x - 2}$

34. $g(x) = \frac{x+2}{x^2 - 1}$

35. $h(x) = \frac{(x^2 + 6x + 5)(x+5)}{(x+5)^3(x-1)}$

36. $f(x) = \frac{x^2 - 1}{x^3 - 2x^2 + x}$

37. $f(x) = \frac{-4x^2 + 1}{x^2}$

38. $k(x) = \frac{x^2 + 1}{x^2 - 1}$

39. $q(x) = \frac{x^2 + 2x}{x^2 - 4x - 5}$

40. $F(x) = \frac{x^2 + x}{x^2 - 2x + 4}$

41. $p(x) = \frac{(x+3)(x-3)}{(x-5)(x+4)(x+3)}$

42. $p(x) = \frac{x^3 + 3x^2}{x^4 - 4x^2}$

43. $f(x) = \frac{x^2 - x - 6}{x-2}$

44. $k(x) = \frac{x^2 + x - 2}{x}$

45. $Q(x) = \frac{4x^2 + 4x - 5}{2x - 5}$

46. $K(x) = \frac{3x^2 - 12x + 15}{3x + 6}$

47. $f(x) = \frac{x^2 - 2}{x-1}$

48. $p(x) = \frac{x^3 + 8}{x+1}$

49. $q(x) = \frac{x^3 + 2x^2 - 2x - 2}{x^2 - 2x + 1}$

50. $f(x) = \frac{x^4 - 1}{x^2}$

Exer. 33–36: Sketch the graph of f .

1 $f(x) = \frac{3}{x-4}$

8 $f(x) = \frac{-3}{x+3}$

2 $f(x) = \frac{-3x}{x+2}$

10 $f(x) = \frac{4x}{2x-5}$

3 $f(x) = \frac{4x-1}{2x+3}$

12 $f(x) = \frac{5x+3}{3x-7}$

4 $f(x) = \frac{(4x-1)(x-2)}{(2x+3)(x-2)}$

14 $f(x) = \frac{(5x+3)(x+1)}{(3x-7)(x+1)}$

5 $f(x) = \frac{x-2}{x^2-x-6}$

16 $f(x) = \frac{x+1}{x^2+2x-3}$

6 $f(x) = \frac{-4}{(x-2)^2}$

18 $f(x) = \frac{2}{(x+1)^2}$

7 $f(x) = \frac{x-3}{x^2-1}$

20 $f(x) = \frac{x+4}{x^2-4}$

9 $f(x) = \frac{2x^2-2x-4}{x^2+x-12}$

22 $f(x) = \frac{-3x^2-3x+6}{x^2-9}$

11 $f(x) = \frac{-x^2-x+6}{x^2+3x-4}$

24 $f(x) = \frac{x^2-3x-4}{x^2+x-6}$

13 $f(x) = \frac{3x^2-3x-36}{x^2+x-2}$

26 $f(x) = \frac{2x^2+4x-48}{x^2+3x-10}$

15 $f(x) = \frac{-2x^2+10x-12}{x^2+x}$

28 $f(x) = \frac{2x^2+8x+6}{x^2-2x}$

17 $f(x) = \frac{x-1}{x^3-4x}$

30 $f(x) = \frac{x^2-2x+1}{x^3-9x}$

19 $f(x) = \frac{-3x^2}{x^2+1}$

32 $f(x) = \frac{x^2-4}{x^2+1}$

Exer. 33–36: Find the oblique asymptote, and sketch the graph of f .

33 $f(x) = \frac{x^2-x-6}{x+1}$

34 $f(x) = \frac{2x^2-x-3}{x-2}$

35 $f(x) = \frac{8-x^3}{2x^2}$

36 $f(x) = \frac{x^3+1}{x^2-9}$

Exer. 37–44: Simplify $f(x)$, and sketch the graph of f .

37 $f(x) = \frac{2x^2+x-6}{x^2+3x+2}$

38 $f(x) = \frac{x^2-x-6}{x^2-2x-3}$

39 $f(x) = \frac{x-1}{1-x^2}$

40 $f(x) = \frac{x+2}{x^2-4}$

41 $f(x) = \frac{x^2+x-2}{x+2}$

42 $f(x) = \frac{x^3-2x^2-4x+8}{x-2}$

43 $f(x) = \frac{x^2+4x+4}{x^2+3x+2}$

44 $f(x) = \frac{(x^2+x)(2x-1)}{(x^2-3x+2)(2x-1)}$

Exer. 45–48: Find an equation of a rational function f that satisfies the given conditions.

- 45 vertical asymptote: $x = 4$
horizontal asymptote: $y = -1$
 x -intercept: 3

- 46 vertical asymptotes: $x = -2, x = 0$
horizontal asymptote: $y = 0$
 x -intercept: 2; ~~$f(2) = 1$~~

- 47 vertical asymptotes: $x = -3, x = 1$
horizontal asymptote: $y = 0$
 x -intercept: -1 ; $f(0) = -2$
hole at $x = 2$

- 48 vertical asymptotes: $x = -1, x = 3$
horizontal asymptote: $y = 2$
 x -intercepts: $-2, 1$; hole at $x = 0$

- 49 A container for radioactive waste. A cylindrical container for storing radioactive waste is to be constructed from lead. This container must be 6 inches thick. The volume of the outside cylinder shown in the figure is to be 16π ft³.